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STABILITY OF RELATIVE MOTION OF PHASES IN TWO-PHASE FLOWS

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The stability of homogeneous states of a two-phase medium in relation to small disturbances (the problem of the correctness of the Cauchy problem for equations of two-phase media) is examined. We show that a consideration of the effect of particle (bubble) diffusion caused by relative motion of the phases is of fundamental importance. The pressure in the disperse phase is a subsidiary factor. The critical stability loss curve is obtained.

The problem of stability of two-phase media has been examined in many papers [1-5]. Existing theories predict a short-wave instability of sedimenting suspensions, fluidized beds, and layers of liquid with bubbles. This instability should lead to the rapid appearance of inhomogeneities within the medium and to the practical unattainability of the homogeneous state. Contradictory to theory, however, manifestly stable states are obtained in experiments [4]. Stability of a liquid with bubbles has been obtained only in [6, 7]. In [6] stability was secured by the action of electrical forces. In the problem of thermocapillary motion in a gas—liquid mixture stability in the short-wave region is obtained by bubble diffusion [7].

1. Equations and Method of Solution

The equations for the change of momentum and conservation of mass of a two-phase medium have the form [1]

$$\varepsilon\rho d\mathbf{v}/dt = \varepsilon\rho\mathbf{g} - \Delta p - \operatorname{div}\mathbf{P}_1 - c\mathbf{F},\ \rho_s\, d\mathbf{u}/dt = \rho_s\mathbf{g} - (1/c)\operatorname{div}\ \mathbf{P}_s + \mathbf{F}; \tag{1.1}$$

$$\partial c/\partial t + \operatorname{div} c\mathbf{u} = 0,\ \partial\varepsilon/\partial t + \operatorname{div}\varepsilon\mathbf{v} = 0,\ c + \varepsilon = 1, \tag{1.2}$$

where ρ, ε, $\mathbf{v}$ and ρ_c, c, $\mathbf{u}$ are the densities, volume concentrations, and densities, respectively, of the carrier and disperse phases; g is the acceleration of gravity. The force of phase interaction F depends, in particular, on the relative velocity of the phases $\mathbf{w} = \mathbf{u} - \mathbf{v}$. The dispersed particles are assumed to be spheres of the same radius R.

At low Reynolds numbers (Re = wR/ν) the force of phase interaction, with due allowance for particle diffusion [8], has the form

$$\mathbf{F} = -\rho\mathbf{g}' + \mathbf{F}^*,\ \mathbf{F}^* = \mathbf{F}_0^* + \mathbf{F}_1^*, \tag{1.3}$$
$$\mathbf{F}_0^* = -\frac{\mu G}{R^2}\mathbf{w},\ \mathbf{F}_1^* = -\frac{\mu}{R^2}\frac{\partial Gw}{\partial w}c^{-1}\mathbf{D}\nabla c,$$

where $\rho\mathbf{g}'$ is the effective repulsive force, $\mathbf{g}' = \mathbf{g} - d\mathbf{v}/dt$; $\mathbf{F}_0^*$ is the viscous resistance force; $\mathbf{F}_1^*$ is the small contribution due to diffusion; G is a dimensionless number; $\mathbf{D}$ is the diffusion tensor:

$$D_{ij} = R|w|(f_{\perp}\delta_{ij} + (f - f_{\perp})w_iw_j/w^2). \tag{1.4}$$

The particle pressure in the medium is given by the tensor

$$(P_s)_{ij} = \rho_s w^2 S_{\perp}\delta_{ij} + \rho_s(S - S_{\perp})w_iw_j. \tag{1.5}$$

At finite Re the coefficient G in (1.3) depends on Re and is connected with the drag coefficient: C_W = G/Re. In this case it is essential to take into account the added-mass effect, which, following [1], we write in model form

$$\mathbf{F}_m = (1/2)\rho(d\mathbf{v}/dt - d\mathbf{u}/dt). \tag{1.6}$$

The fluctuations of the acceleration of the liquid must also be taken into account:

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$$\Sigma_{ij} = \rho w^2 \sigma_- \delta_{ij} + \rho w_i w_j (\sigma - \sigma_\perp), \quad (1.7)$$

and the force **F** will then take the form

$$\mathbf{F} = -\rho \mathbf{g}' + \mathbf{F}_0^* + \mathbf{F}_1^* + \mathbf{F}_m + \operatorname{div}\boldsymbol{\Sigma}, \quad (1.8)$$
$$\mathbf{F}_1^* = \partial \mathbf{F}_0^* / \partial w_k c^{-1} \, (\mathbf{D}\nabla c)_k.$$

The recurrent subscripts in (1.8) mean summation. The dimensionless quantities G, f, S, and σ, contained in Eqs. (1.3)-(1.8), in the general case depend on ε, Re, and Fr. The Froude number $\mathrm{Fr} = w/\sqrt{Rg}$.

At the small Fr limit the tensor **D** is nonzero, and the contributions of the pressure $\mathbf{P}_s$ (1.5) and $\boldsymbol{\Sigma}$ (1.7) to the equations are negligible:

$$\mathbf{D} \neq 0, \ \mathbf{P}_s = 0, \ \boldsymbol{\Sigma} = 0 \text{ when } \mathrm{Fr} = 0. \quad (1.9)$$

At the large Fr limit, which is possible when $\rho_s \gg \rho$, the contributions of the pressure $\mathbf{P}_s$, $\boldsymbol{\Sigma}$, and the diffusion tensor to the equations tend to zero [8]:

$$S, \ \boldsymbol{\Sigma} \sim 1/\mathrm{Fr}^2, \ f \sim 1/\mathrm{Fr} \text{ when } \mathrm{Fr} \to \infty. \quad (1.10)$$

This can be attributed to the reduction of random motions when $\mathrm{Fr} \gg 1$.

We note that at low Re ($\ll 1$) in a rarefied system ($c \ll 1$) $S \sim c^{5/3}$ and $f \sim 1$ in order of magnitude. When $\mathrm{Re} \ll 1$, Σ and $\mathbf{F}_m$ can always be neglected in the expression for the force (1.8), and (1.3) will be valid.

The problem consists in determination of the stability of the homogeneous solution of Eqs. (1.1) with the pressure in the particle medium given by (1.5) and the interphase force given by (1.3) or (1.8):

$$c = 1 - \varepsilon = \text{const}, \ \mathbf{v} = -\mathbf{w} = \text{const } \mathbf{e}, \ \mathbf{u} = 0, \ p = -(\varepsilon\rho + c\rho_s) g z. \quad (1.11)$$

Here z is the coordinate in the direction of gravitational acceleration **e**. The velocity **w** in the homogeneous state is given by the formulas

$$G\mathbf{w} = -(\rho_s - \rho) \mathbf{g} R^2/\mu, \ C_w |\mathbf{w}| \mathbf{w} = (\rho_s/\rho - 1) \ \mathbf{g} R. \quad (1.12)$$

As a result of this, the Froude number is given by

$$\mathrm{Fr}^2 = |(\rho_s/\rho - 1)(\mathrm{Re}/G)| = |\rho_s/\rho - 1| C_w^{-1}. \quad (1.13)$$

When $\mathrm{Re} \gg 1$ the drag coefficient C_w depends weakly on Re and, hence, as (1.13) shows, it is convenient to introduce another dimensionless parameter ($\chi = \rho_s/\rho$) instead of Fr.

A method of solving the problem of instability of the homogeneous state (1.11) can be devised by taking into account the asymptotic sense of Eqs. (1.1), (1.5), (1.3), or (1.8), based on the existence of two small parameters:

$$\alpha = (R/wT)\mathrm{Fr}^2 \ll 1, \ \beta = (R/L)\,\mathrm{Fr}^2 \ll 1. \quad (1.14)$$

These inequalities are not strong restrictions, since the continuous-medium approach is suitable only for fairly large scale lengths $L \gg R$ and time $T \gg R/w$. Hence, the stability problem can be solved by an iteration method. The equations of the homogeneous state (1.12) will be the main approximation from which, in particular, we can obtain the main approximation of the small-disturbance equations for small α and β. Gradient terms will appear in this equation as small corrections to the next approximation.

2. Concentration Waves at Low Re and Fr

If $\mathrm{Fr} \ll 1$, then, in view of (1.14), the difference in the phase accelerations is negligible ($d\mathbf{v}/dt \approx d\mathbf{u}/dt$), and the contribution of disturbances of the phase accelerations can be neglected in the small-disturbance equations. Taking (1.9) into account and also the fact that when $\mathrm{Re} \ll 1$, $\mathrm{Fr} \ll 1$, $G(\varepsilon, \mathrm{Re}, \mathrm{Fr}) \approx G(\varepsilon, 0, 0)$, then from (1.3) and (1.1) we obtain $(\rho_s - \rho)\mathbf{g} - G(\mathbf{w} + c^{-1}\mathbf{D}\nabla c) = 0$, from which for small disturbances

$$\mathbf{w}\varepsilon' \partial \ln G / \partial \varepsilon + \mathbf{w}' + c^{-1} \mathbf{D} \nabla c' = 0. \quad (2.1)$$

Here and henceforth a dash denotes small disturbances of the respective parameters. Multiplying the continuity equations (1.2) by −ε and c, respectively, and then adding them, we obtain for small disturbances

$$\partial\varepsilon'/\partial t - c\mathbf{w}\nabla\varepsilon' - \varepsilon c \operatorname{div} \mathbf{w}' = 0. \qquad (2.2)$$

Determining div w' from (2.1) and substituting in (2.2) we obtain the small-disturbance equation

$$\partial\varepsilon'/\partial t - c\theta(\mathbf{w}\nabla)\varepsilon' = \varkappa_\perp\Delta_\perp\varepsilon' + \varkappa\partial^2\varepsilon'/\partial z^2,\ \theta = 1 - (\varepsilon/G)dG/d\varepsilon, \qquad (2.3)$$
$$\varkappa = \varepsilon R|\mathbf{w}|f,\ \varkappa_\perp = \varepsilon R|\mathbf{w}|f_\perp,$$

where $\Delta_\perp$ is the Laplace operator in a plane orthogonal to the undisturbed value of the relative velocity w; the coefficients f and $f_\perp$ depend on c; $\varkappa = \varepsilon D$ (D is the diffusion coefficient); $\theta \approx .5$, in view of the known empirical relation [8, 9].

Equation (2.3) is the equation of convective diffusion of disturbances of particle concentration with allowance for anisotropy of the diffusion coefficient. The main feature is the left-hand side of the equation, which describes the motion of disturbances with velocity $-c\theta w$.

Disturbances of length of the order of λ in traversing a distance $\sim\lambda$ are spread over a small value $\Delta z \sim \sqrt{R\lambda}$. The concentration disturbance is appreciably spread at characteristic distances $L \sim \lambda^2/R$. It is obvious that on the large-scale L only short-wave disturbances of length $\lambda < \sqrt{LR}$ manage to relax.

It is apparent that at small Fr a two-phase medium is stable. This conclusion is consistent with experiments on the sedimentation of suspensions of small particles in liquids [9], the results of which correspond to small values of Fr^2 ($\sim 10^{-2}$–10^{-4}). In these experiments only stable states of the medium were actually observed.

3. Stability of Two-Phase Media at Finite Fr and Re

We consider the one-dimensional problem of the dynamics of small disturbances of the homogeneous state (1.11). Linearization of Eqs. (1.1) and (1.8) gives

$$\frac{\mu}{R^2}\delta(Gw) = \frac{\mu}{R}\frac{\partial Gw}{\partial w}\frac{|w|}{c}f\frac{\partial\varepsilon'}{\partial z} - \left(\rho_s + \frac{\rho}{2}\right)\frac{\partial u'}{\partial t} \qquad (3.1)$$
$$+ \frac{1}{c}\frac{\partial}{\partial z}\delta[(\rho\sigma - \rho_s S)w^2] + \frac{3}{2}\rho\left(\frac{\partial v'}{\partial t} - w\frac{\partial v'}{\partial z}\right),$$
$$\delta = \varepsilon'\frac{\partial}{\partial\varepsilon} + w'\frac{\partial}{\partial w},\ \delta(Gw) = \frac{\partial Gw}{\partial w}\left(w' - \frac{\theta - 1}{\varepsilon}w\varepsilon'\right).$$

The left-hand side of (3.1) is the main approximation for small parameters α and β (1.14). In this approximation $\delta(Gw) = 0$, from which it follows that

$$w' = \frac{\theta - 1}{\varepsilon}w\varepsilon',\ \theta = 1 - \varepsilon\frac{\partial G}{\partial\varepsilon}\left(\frac{\partial Gw}{\partial w}\right)^{-1}. \qquad (3.2)$$

From (3.2) we find $\partial w'/\partial z$ and substitute it in (2.2). Then, in the main approximation we obtain the equation

$$\partial\varepsilon'/\partial t - c\theta w\partial\varepsilon'/\partial z = 0. \qquad (3.3)$$

To obtain Eq. (3.3) in the next approximation for small α and β we must express $\partial w'/\partial z$ by ε' in this approximation. For this we express u' and v' in (3.1) in terms of ε' and w' with the aid of the integral for the one-dimensional continuity equations: $u' = \varepsilon' w$, $v' = -w' + \varepsilon' w$. The disturbance w' can be eliminated from the right-hand side of (3.1) after expressing w' in terms of ε' and its derivatives with the aid of (3.2). The derivative of ε' with respect to time can be expressed as a derivative with respect to z by means of (3.3). Thus, from (3.1) we find, with accuracy to terms of the second order of smallness in α and β:

$$\frac{\partial w}{\partial z} = \frac{\theta - 1}{\varepsilon}w\frac{\partial\varepsilon'}{\partial z} + \frac{\varkappa}{\varepsilon c}\frac{\partial^2\varepsilon'}{\partial z^2}.$$

Substituting this expression in (2.2) we obtain an equally accurate equation for ε'

$$\frac{\partial\varepsilon'}{\partial t} - c\theta w\frac{\partial\varepsilon'}{\partial z} = \varkappa\frac{\partial^2\varepsilon'}{\partial z^2}; \qquad (3.4)$$

$$\varkappa = \varepsilon R\,|w|\left[f - \frac{\mathrm{Fr}^2}{|1-\rho/\rho_s|}\,a\,(c^2\theta^2 + S^* + H + H_m)\right], \tag{3.5}$$

$$a = G\,(\partial Gw/\partial w)^{-1},\ S^* = \left(\frac{1}{w^2}\frac{\partial}{\partial\varepsilon} + \frac{\theta-1}{\varepsilon w}\frac{\partial}{\partial w}\right)\left[w^2\left(S - \frac{\rho}{\rho_s}\sigma\right)\right],$$

$$H = \frac{\rho}{\rho_s}\frac{c}{\varepsilon}(1-c\theta)^2,\ H_m = \frac{\rho}{\rho_s}\frac{c}{2\varepsilon}[(1-c\theta)^2 + c\varepsilon\theta^2].$$

The term f in (3.5) is due to particle diffusion with diffusion tensor $D \sim R|w|f$; the term containing $c^2\theta^2$ is due to the inertial force $\rho_s du/dt$; S^* is the particle pressure P_s and the tensor Σ in the force **F**. Finally, the terms in (3.5) proportional to H and H_m are due respectively to the effective repulsive force $-\rho g'$ and the force $\mathbf{F}_m$ representing the added-mass effect.

When $\varkappa > 0$ the Cauchy problem for Eq. (3.5) is correct and the system is stable. The dynamics of the disturbances is similar to the case analyzed in Sec. 2. If $\varkappa < 0$, the system is unstable. The Cauchy problem is incorrect. Disturbances with the smallest wavelength grow most rapidly. For any macroscopic scale L we can indicate a length $\lambda \ll L$ (but $\lambda \gg R$) such that disturbances of scale λ can increase greatly in amplitude before they are removed from the system at rate $-c\theta w$.

It is significant that instability at $\rho_s \gg \rho$, as (3.5) shows, can be appreciable only in fairly concentrated systems, since the increment is proportional to c^2.

From the form of coefficient $\varkappa$ in (3.5) we can draw the following important conclusions.

1. Stability can be secured only by particle diffusion, the presence of the disperse phase pressure P_s, and fluctuations of the effective repulsive force Σ. The diffusion effect always promotes stability (f > 0), whereas the pressure and fluctuations of the unsteady interaction force may lead also to instability, since the sign of S^* in (3.5) may be different. We can have the situation in which these effects stabilize the system ($S^* < 0$) in one region of concentrations, and promote instability ($S^* > 0$) in another region.

2. The role of the disperse phase pressure is insignificant when Re $\ll$ 1 and the phase densities are comparable ($\rho_s \sim \rho$). The stability of the system ($\varkappa > 0$) in this case can be attributed entirely to the particle diffusion mechanism.

3. The system is always unstable at high Fr ($\gg$1), since, in correspondence with (1.10), the coefficients f and $S^* \to 0$, and Fr $\to \infty$. When Fr $\to \infty$ the density ratio ρ_s/ρ always approaches infinity.

4. There is a critical stability loss curve $\mathrm{Fr} = \mathrm{Fr}^\circ(\mathrm{Re}, \varepsilon)$ or $\rho_s/\rho = \chi^\circ(\mathrm{Re}, \varepsilon)$, since the coefficient $\varkappa$ (3.5) is a function of Fr, Re, and ε, or the equivalent variables ρ_s/ρ, Re, and ε. When $\mathrm{Fr} > \mathrm{Fr}^\circ$ or $\rho_s/\rho > \chi^\circ$ the system is unstable.

5. Stability loss occurs at finite values of Fr° ($\sim$1). In this case the accuracy of the method proposed for two-phase media in [8] is equal to that of the continuous-medium method, as conditions (1.14) show. In the region Fr $\to \infty$, where the method of [8] becomes inaccurate, there are no stable homogeneous states of two-phase media. Destruction of homogeneous states when Fr $\gg$ 1 occurs in the short-wave region in a small time $\tau \sim (R/w)\cdot(\varepsilon c\theta \mathrm{Fr})^{-2}$, if $\varepsilon c\theta\,\mathrm{Fr} < 1$, and in time $\tau \sim R/w$, if the opposite is the case.

4. Limits of Small and Large Re

When Re $\ll$ 1 the coefficient $\varkappa$ in (3.4) is

$$\varkappa = (f - \mathrm{Fr}^2(c^2\theta^2 + S^*))\varepsilon R|w|, \qquad \mathrm{Fr}^2 = (\mathrm{Re}/G)\rho_s/\rho, \tag{4.1}$$

where S^* depends only on the disperse phase pressure. The term with Fr^2 in (4.1) is significant only when $\rho_s \gg \rho$. The quantities f and S^* depend only on Fr and ε and, hence, the condition $\varkappa = 0$ determines the stability curve

$$\mathrm{Fr} = \mathrm{Fr}^0(\varepsilon). \tag{4.2}$$

In view of (1.9) and (1.10), when Fr $\ll$ 1 the medium is stable, and when Fr $\gg$ 1 it is unstable. We must point out that in the considered limiting case the stability boundary is independent of the density ratio ρ_s/ρ.

In formula (3.5) at large Re ($\gg 1$) the coefficient $a = 2$ and the coefficients S, σ, and θ depend very weakly on Re. Hence, there must be a limiting stability curve

$$\mathrm{Fr} = \mathrm{Fr}^0(\varepsilon) \quad \text{or} \quad \rho_s/\rho = \chi^0(\varepsilon) \ \text{when Re} \gg 1. \tag{4.3}$$

The first formula in (4.3) has the same form as (4.2), and the second way of writing it is suitable only at large Re.

The disperse phase pressure P_s probably plays a minor role in comparison with diffusion not only when Re $\ll 1$, but also when Re $\gg 1$. This is indicated by the results of experiments [10] on homogeneous fluidization by a liquid at Re ~ 100. Treatment of these data shows that in the range $\varepsilon = 0.53$–0.7 the coefficients in the formula for the pressure P_s (1.5) are very small: $S \approx 0.01$, $S_\perp \approx 0.005$. The coefficients σ and $\sigma_\perp$ in the tensor $\mathbf{\Sigma}$, contained in $\mathbf{F}$, in this case have values that are probably an order greater than S and $S_\perp$.

5. Comparison of Theory and Experiment

The results obtained enable us to explain the essential features of sedimentation of suspensions, fluidized beds, and layers of liquids with bubbles. A fluidized bed is formed if the velocity of an ascending flow of gas or liquid through a stationary layer of solid particles is sufficiently high. When the velocity of the liquid increases the bed expands.

In experiments and in practice [4] homogeneous and highly inhomogeneous states of fluidized beds are observed. The characteristic feature of the latter state is that, despite the most uniform and homogeneous flow of gas through the bottom of the bed (e.g., through a fine-pored metal plate [4]), large-scale inhomogeneities are spontaneously produced within the layer. An inhomogeneous state of a fluidized bed in which there are great irregularities of particle concentration can be interpreted as the result of development of instability of the homogeneous state.

The theory of stability of relative motion of the phase provides the first explanation of the following experimental manifestations of the existence of homogeneous and inhomogeneous states of fluidized beds.

1. At finite Re ($\gtrsim 1$) for solid particles suspended in gas only inhomogeneous states occur and the stable homogeneous state is not observed [4]. The theoretical explanation of this is that the instability condition is fulfilled, since at high particle density Fr $\gg 1$.

2. At finite Re ($\gtrsim 1$) the homogeneous state of a fluidized bed is observed only when the phase densities are close ($\rho_s \lesssim \rho$) [4, 10].

3. An increase in particle density or reduction of the viscosity of the carrier medium (Re < 1) always promotes inhomogeneity of fluidization. In particular, in the case of fluidization by liquids the homogeneous state is observed much more frequently than in the case of fluidization by gases [4]. The theoretical explanation of this is that with increase in density ρ or reduction of viscosity Fr increases and approaches the critical value Fr^0.

4. An increase in radius R of the particles suspended in the flow leads to the appearance of inhomogeneities in experiments [4]. This can also be attributed to an increase in Fr.

5. Many experiments (see [4, 11], for instance), beginning with those in [12], have shown that at large values of the parameter $j^2/(gd) \gg 1$ (j is the velocity of gas through unit cross section of the bed, d is the particle diameter) an exceptionally inhomogeneous state of a fluidized bed is formed. Hitherto there has been no explanation of this experimental fact. Attempts to explain it by introducing incorrect ideas regarding the buoying up of bubbles of radius of the same order as that of the particles [11] were unsatisfactory.

The correct explanation of this empirical fact is that at large Fr ($\gg 1$) [$\mathrm{Fr}^2 = 2(1-c)^{-2} j^2/(gd)$] a heterogeneous continuous medium becomes unstable in the short-wave region and ceases to be macroscopically homogeneous.

In experimental investigations [13-16] the critical parameters of the transition of a fluidized bed of solid particles in a gas flow from the homogeneous to the inhomogeneous

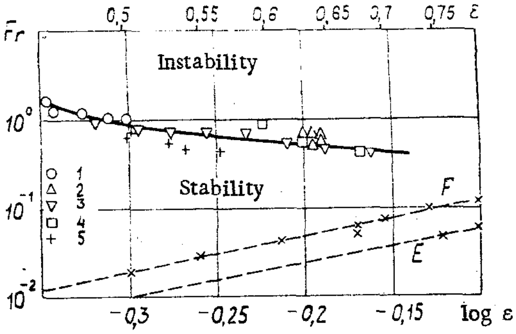

Fig. 1

state were measured. This transition occurred at a certain gas flow velocity, when splashes were observed relatively suddenly on the surface of the bed and the height of the bed started to decrease rapidly with increase in velocity.

An analysis of the experimental data of [13-16] from the standpoint of the above-developed stability theory is shown in Fig. 1. The experimental results are represented in coordinates Fr and $\varepsilon = 1 - c$. Points 1 and 2 correspond to [13], and points 3-5 to [14-16]. It is apparent that the experimental results correspond well with the existence of the single stability curve (4.2). All the presented results, except those of [16], correspond to particles of a narrow fraction. A considerable number of the points relate to significantly nonspherical particles, which leads to some spread of the points. The experimental data of [16] lie within the stability limits and, hence, points 5 in Fig. 1 lie within the region of stable states.

It should be noted that experiments with broad fractions lead to differing stability curves $Fr^{\circ}(c)$, since the stability naturally depends on the particle size distribution. The characteristic values of Fr° at which stability is lost are of the order of 1 in all the experiments.

Figure 1 also shows the results of treating suspension sedimentation data [9]. Line F corresponds to sedimentation of glass spheres of diameter 0.01 cm in water, and line E to the sedimentation of particles of mean diameter 0.0096 cm and density $\rho_s = 1.88$ g/cm^3 in a liquid of density $\rho \approx 1$ g/cm^3 and viscosity $\mu \approx 0.01$ g/cm·sec. It is apparent that stable sedimentation of the suspensions occurs in the region $Fr < Fr^{\circ}$ in complete agreement with the theory.

For a liquid with suspended bubbles the theory indicates the possibility of stable homogeneous states, since Fr is always $\leqslant 1$. At low Re a layer of liquid with bubbles is always stable, as follows from points 2 and 3. At finite Re a liquid with bubbles corresponds to the most stable situation $\rho_s/\rho = 0$. It is known that even when $\rho_s > \rho$ stable homogeneous states exist [4, 10]. Homogeneous (on the average) states of concentrated systems of suspended bubbles have been observed in experiments [17, 18].

We note in conclusion that the existence of a two-phase medium as a homogeneous continuous medium is ensured by random motions due to hydrodynamic interaction. The entire stability theory is contained within the framework of applicability of the method of representing a two-phase medium by consideration of random motions [8].

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PROBLEM OF FIRING PLANAR NOZZLES IN SHOCK TUBES

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UDC 533.6

Numerous recent investigations are concerned with studying the propagation of shock waves in channels with variable cross sections. There is no rigorous description of all details of such flow, so that each investigation is carried out based on a chosen simplified model. In particular, in order to analyze processes related to firing nozzles in shock tubes, flow models taking into account the passage of the primary shock wave along the nozzle, the contact surface, the secondary shock wave, and nonstationary rarefaction waves are widely used [1]. Such models permit determining the trajectory of the shock waves, which in many cases [1-4] coincide with the experimentally observed trajectories, although the viscosity of the gas and the two-dimensional nature of the flow were not taken into account in the calculations. The effects indicated are manifested most strongly in the supersonic part of the nozzle, near its walls, when the secondary shock wave interacts with the boundary layer, causing separation of the flow [1, 5, 6]. At the present time, there is no clear idea of how the flow separation affects the flow parameters and the continuance of firing, measured through the lateral walls of the planar nozzle. The possibilities of computational methods are limited due to the absence of criteria for separation in a nonstationary flow and spread of separation data in stationary flows [1]. Also, the relation between the flow separation from diverging and from parallel walls of the nozzle is not clear. We note that when the flow is visualized optically [1, 5], the flow separation from the diverging walls is clearly manifested, but the effects are not observed on the parallel walls due to the small optical thickness of the imhomogeneities. At the same time, the schemes for measuring the optical amplification are more sensitive to the effects on the parallel

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